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APPENDIX B:

Technology Center 2100

CLEAN VERSION OF ALL PENDING CLAIMS AS NOW PRESENTED

(Three Times Amended) A method for establishing cryptographic communications, comprising the steps of: encoding a plaintext message word M to a ciphertext word C, wherein M corresponds to a number representative of a message and wherein 4 5 $0 \le M \le n-1$, wherein n is a composite number formed by the product of p₁•p₂•...•p_k, k is an integer 6 greater than 2 and $p_1, p_2, ..., p_k$ are distinct random prime numbers, C is a number 7 representative of an encoded form of message word M, and wherein said encoding step 8 comprises transforming said message word M to said ciphertext word C, whereby 9 $C \equiv M^e \pmod{n}$, 10 and wherein e is a number relatively prime to (p₁-1), (p₂-1), ..., and (p_k-1); and 11 decoding said ciphertext word C to a receive message word M', said decoding step being 12 performed using a decryption exponent d that is defined by 13 $d \equiv e^{1} \mod ((p_1-1)(p_2-1)...(p_k-1)),$ 14 said decoding step including the further steps of, 15 defining\a plurality of k sub-tasks in accordance with 16 $M_1' \equiv C_1^{d_1} \pmod{p_1},$ 17 $M_2' \equiv C_2^{d_2} \pmod{p_2},$ 18 19 $M_k' \equiv C_k^{d_k} \pmod{p_k}$ 20 wherein 21 $C_1 \equiv C \pmod{p_1}$ 22 $C_2 \not\equiv C \pmod{p_2}$, 23 24 $C_k \equiv C \pmod{p_k}$, 25 26

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 $d_1 \equiv d \pmod{(p_1 - 1)},$

 $d_2 \equiv d \pmod{(p_2 - 1)}, \text{ and}$

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 $d_k \equiv d \pmod{(p_k-1)},$

solving said sub-tasks to determine results M_1 , M_2 , ... M_k , and

combining said results of said sub-tasks to produce said receive message word M',

- 33 wherein M'=M.
 - 1 18. A method as recited in claim 17 wherein said step of combining said results of said sub-
- 2 tasks includes a step of performing a recursive combining process to produce said receive
- 3 message word M'.

in accordance with

- 1 19. A method as recited in claim 18 wherein said recursive combining process is performed
- 3 $Y_i \equiv Y_{i-1} + [(M_i' Y_{i-1}) (w_i^{-1} \mod p_i) \mod p_i] \cdot w_i \mod n,$
- 4 wherein $2 \le i \le k$, and
- 5 $M = Y_k, Y_1 = M_1, \text{ and } W_i = \prod_{j < i} p_j.$
- 1 20. A method as recited in claim 17 wherein said step of combining said results of said sub-
- 2 tasks includes a step of performing a summation process to produce said receive message word
- 3 M'.
- 1 21. A method as recited in claim 20 wherein said summation process is performed in
- 2 accordance with
- $M' \equiv \sum_{i=1}^{k} M'_{i}(w_{i}^{-1} \operatorname{mod} p_{i})w_{i} \operatorname{mod} n,$
- 4 where
- $\mathbf{w_i} = \prod_{i \neq i} p_j.$

1	22. (Three Times Amended) A cyptographic communications system for establishing
2	communications, comprising:
3	a communication medium;
4	encoding means coupled to said communication medium and adapted for transforming a
5	transmit message word M to a ciphertext word C and for transmitting said ciphertext word C on
6	said medium, wherein M corresponds to a number representative of a message, and
7	$0 \le M \le n-1$, wherein n is a composite number of the form,
8	$\mathbf{n} = \mathbf{p}_1 \bullet \mathbf{p}_2 \bullet \dots \bullet \mathbf{p}_k$
9	wherein k is an integer greater than 2 and p ₁ , p ₂ ,,p _k are distinct random prime
10	numbers, and wherein said ciphertext word C corresponds to a number representative of an
11	enciphered form of said message word M and corresponds to
12	$C \equiv M^{\epsilon} \pmod{n},$
13	wherein e is a number relatively prime to (p_1-1) , (p_2-1) ,, and (p_k-1) ; and
14	decoding means communicatively coupled with said communication medium for
15	receiving said ciphertext word C via said medium, said decoding means being operative to
16	perform a decryption process for transforming said ciphertext word C to a receive message word
17	M', wherein M' corresponds to a number representative of a deciphered form of C, said
18	decryption process using a decryption exponent d that is defined by
19	$d \equiv e^{-1} \mod ((p_1-1) (p_2-1) \setminus (p_k-1)),$
20	said decryption process including the steps of
21	defining a plurality of k sub-tasks in accordance with
22	$M_1' \equiv C_1^{d_1} \pmod{p_1},$
23	$M_2' \equiv C_2^{d_2} \pmod{p_2},$
24	
25	$M_k' \equiv C_k^{d_k} \pmod{p_k},$
26	wherein
27	$C_1 \equiv C \pmod{p_1},$
28	$C_2 \equiv C \pmod{p_2},$
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 $C_k \equiv C \pmod{p_k},$

31

 $d_1 = d \pmod{(p_1 - 1)},$

33 $d_2 \neq d \pmod{(p_2-1)}$,

34

 $d_k \equiv d \pmod{(p_k - 1)},$

solving said sub-tasks to determine results M₁', M₂', ... M_k', and

combining said results of said sub-tasks to produce said receive message word M'

38 whereby M'=M.

- 1 23. A cyptographic communications system as recited in claim 22 wherein said decoding
- 2 means is operative to combine said results of said sub-tasks by performing a recursive combining
- 3 process to produce said receive message word M'.
- 1 24. A cyptographic communications system as recited in claim 23 wherein said decoding
- 2 means is operative to perform said recursive combining process in accordance with
- 3 $Y_i \equiv Y_{i-1} + [(M_i' Y_{i-1}) (w_i^{-1} \mod p_i) \mod p_i] \cdot w_i \mod n$
- 4 wherein $2 \le i \le k$, and
- 5 $M = Y_k, Y_1 = M_1, \text{ and } W_i = \prod_{j < i} p_j$
- 1 25. A cyptographic communications system as recited in claim 22 wherein said decoding
- 2 means is operative combine said results of said sub-tasks by performing a summation process to
- 3 produce said receive message word M'.
- 1 26. A cyptographic communications system as recited in claim 25 wherein said decoding
- 2 means is operative to perform said summation process accordance with

$$M \equiv \sum_{i=1}^{k} M_i (w_i^{-1} \mod p_i) w_i \mod n,$$

where

$$\mathbf{w}_{i} = \prod_{j \in I} p_{j}.$$

1 27. (Three Times Amended) A method for establishing cryptographic communications,

2 comprising the step of:

encoding a plaintext message word M to a ciphertext word C, wherein M corresponds to a number representative of a message, and

 $5 0 \le M \le n-1,$

6 n being a composite number formed from the product of p₁•p₂•...•p_k, wherein k is an integer

7 greater than 2 and $p_1, p_2, ..., p_k$ are distinct random prime numbers, and wherein the ciphertext

word C is a number representative of an encoded form of message word M, wherein said step of

9 encoding includes the steps of

defining a plurality of k sub-tasks in accordance with

$$C_1 \equiv M_1^{e_1} \pmod{p_1},$$

12
$$C_2 \equiv M_2^{e_2} \pmod{p_2},$$

13

3

4

8

10

$$C_k \equiv M_k^{e_k} \pmod{p_k},$$

15 where

$$16 M_1 \equiv M \pmod{p_1},$$

$$M_2 \equiv M \pmod{p_2},$$

18

$$19 M_k \equiv M \pmod{p_k},$$

20

21
$$e_1 \equiv e \pmod{(p_1 - 1)},$$

22
$$e_2 \equiv e \pmod{(p_2 - 1)}$$
, and

$$e_k \equiv e \pmod{(p_k - 1)},$$

- wherein e is a number relatively prime to (p_1-1) , (p_2-1) , ..., and (p_k-1) ,
- solving said ub-tasks to determine results C1, C2, ... Ck, and
- 27 combining said results of said sub-tasks to produce said ciphertext word C.
 - 1 28. A method as recited in claim 27 wherein said step of combining said results of said sub-
 - 2 tasks includes a step of performing a recursive combining process to produce said ciphertext
 - 3 word C.
 - 1 29. A method as recited in claim 28 wherein said recursive combining process is performed
 - 2 in accordance with
 - $Y_{i} \equiv Y_{i-1} + [(C_{i} \setminus Y_{i-1}) (w_{i}^{-1} \mod p_{i}) \mod p_{i}] \cdot w_{i} \mod n,$
 - 4 wherein $2 \le i \le k$, and
 - 5 $C = Y_k, Y_1 = C_1, \text{ and } W_i = \prod_{j < i} p_j.$
 - 1 30. A method as recited in claim 27 wherein said step of combining said results of said sub-
 - 2 tasks includes a step of performing a summation process to produce said ciphertext word C.
 - 1 31. A method as recited in claim 30 wherein said summation process is performed in
 - 2 accordance with
 - $C \equiv \sum_{i=1}^{k} C_i (w_i^{-1} \mod p_i) w_i \mod n$
 - 4 where
 - $\mathbf{w}_{i} = \prod_{j \neq i} p_{j}.$
 - 1 32. (Three Times Amended) A cyptographic communications system for establishing
 - .2 communications, comprising:
 - a communication medium;

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encoding means coupled to said communication medium and operative to transform a
4
      transmit message word M to a ciphertext word C, and to transmit said ciphertext word C on said
 5
 6
      medium, wherein M corresponds to a number representative of a message, and
 7
              0 \le M \le n-1,
      n being a composite number formed from the product of p_1 \cdot p_2 \cdot ... \cdot p_k wherein k is an integer
 8
      greater than 2 and p_1, p_2, ... \setminus p_k, are distinct random prime numbers, and wherein the ciphertext
 9
      word C is a number representative of an encoded form of message word M, said encoding means
10
      being operative to transform said transmit message word M to said ciphertext word C by
11
      performing an encoding process comprising the steps of
12
               defining a plurality of k sub-tasks in accordance with
13
                               C_1 \equiv M_1^{e_1} \pmod{p_1},
14
                               C_2 \equiv M_2^{e_2} \pmod{p_2},
15
16
                               C_k \equiv M_k^{e_k} \pmod{p_k},
17
18
                       where
                               M_1 \equiv M \pmod{p_1},
19
                               M_2 \equiv M \pmod{p_2},
20
21
                               M_k \equiv M \pmod{p_k},
22
23
                               e_1 \equiv e \pmod{(p_1 - 1)},
24
                                e_2 \equiv e \pmod{(p_2 - 1)}, and
25
26
                               e_k \equiv e \pmod{(p_k - 1)},
27
                       wherein e is a number relatively prime to (p_1-1), (p_2-1), ..., and (p_k-1),
28
               solving said sub-tasks to determine results C<sub>1</sub>, C<sub>2</sub>,... C<sub>k</sub>, and
29
               combining said results of said sub-tasks to produce said ciphertext word C.
30
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- 1 33. A cyptographic communications system as recited in claim 32 wherein said encoding
- 2 means is operative to dombine said results of said sub-tasks by performing a recursive combining
- 3 process to produce said ciphertext word C.
- 1 34. A cyptographic communications system as recited in claim 33 wherein said encoding
- 2 means is operative to perform said recursive combining process in accordance with
- 3 $Y_i \equiv Y_{i-1} + [(C_i Y_{i-1}) (w_i^{-1} \mod p_i) \mod p_i] \cdot w_i \mod n,$
- 4 wherein $2 \le i \not \models k$, and
- 5 $C = Y_k, Y_1 = C_1 \setminus \text{and } W_i = \prod_{j \le i} p_j.$
- 1 35. A cyptographic communications system as recited in claim 32 wherein said encoding
- 2 means is operative to combine said results of said sub-tasks by performing a summation process
- 3 to produce said message word C.
- 1 36. A cyptographic communications system as recited in claim 35 wherein said encoding
- 2 means is operative to perform said summation process in accordance with
- $C \equiv \sum_{i=1}^{k} C_i (w_i^{-1} \mod p_i) w_i \mod n,$
- 4 where
- $\mathbf{w_i} = \prod_{j \neq i} p_j.$
- 1 37. (Three Times Amended) A method for establishing cryptographic communications,
- 2 comprising the steps of:
- decoding a ciphertext word C to a message word M, wherein M corresponds to a number
- 4 representative of a message and wherein
- $5 0 \le M \le n-1$
- 6 wherein n is a composite number formed by the product of phop₂•...•p_k, k is an integer greater
- 7 than 2 and $p_1, p_2, ..., p_k$ are distinct random prime numbers, C is a number representative of an

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encoded form of message word M that is encoded by transforming said message word M to said
 8
      ciphertext word C whereby
 9
              C \equiv M^e \pmod{n},
10
              and wherein e is a number relatively prime to (p_1-1), (p_2-1), ..., and (p_k-1);
11
              said decoding step being performed using a decryption exponent d that is defined by
12
                       d \equiv e^{-1} \mod ((p_1-1)(p_2-1)...(p_k-1)),
13
               wherein said step of decoding includes the steps of
14
                       defining a plurality of k sub-tasks in accordance with
15
                               M_1 \equiv C_1^{d_1} \pmod{p_1},
16
                               M_2 \equiv C_2^{d_2} \pmod{p_2},
17
18
                               M_k \equiv C_k^{d_k} \pmod{p_k},
19
                       wherein
20
                               C_1 \equiv C \pmod{p_1},
21
                               C_2 \equiv C \pmod{p_2}
 22
23
                                C_k \equiv C \pmod{p_k},
 24
 25
                                d_1 \equiv d \pmod{(p_1 - 1)},
 26
                                d_2 \equiv d \pmod{(p_2 - 1)}, and
 27
 28
                                d_k \equiv d \pmod{(p_k - 1)},
 29
                        solving said sub-tasks to determine results M_1, M_2, ... M_k, and
 30
                        combining said results of said sub-tasks to produce said message word M.
 31
                A method as recited in claim 37 wherein said step of combining said results of said sub-
        38.
  1
       tasks includes a step of performing a recursive combining process to produce said message word
  2
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M.

- A method as redited in claim 38 wherein said recursive combining process is performed 39. 1
- in accordance with 2
- $Y_i \equiv Y_{i-1} + [(M_i Y_{i-1}) (w_i^{-1} \mod p_i) \mod p_i] \cdot w_i \mod n$ 3
- wherein $2 \le i \le k$, and 4
- $M = Y_k$, $Y_j = M_1$, and $W_i = \prod_{j < i} p_j$. 5
- A method as recited in claim 37 wherein said step of combining said results of said sub-40. 1
- tasks includes a step of performing a summation process to produce said message word M. 2
- A method as recited in claim 40 wherein said summation process is performed in 41. 1
- accordance with
- $M \equiv \sum_{i=1}^{k} M_i (w_i^{-1} \mod p_i) w_i \mod n,$ where $W_i = \prod_{j \neq i} p_j.$
- 5
- 42. (Three Times Amended) A cyptographic communications system for establishing 1
- 2 communications, comprising:
- a communication medium; 3
- decoding means communicatively coupled with said communication medium for 4
- receiving a ciphertext word C via said medium, and being operative to transform said ciphertext 5
- word C to a receive message word M', wherein a message M corresponds to a number 6
- representative of a message and wherein, 7
- $0 \le M \le n-1$ 8
- wherein n is a composite number formed by the product of p₁•p₂•...•p_k, k is an integer greater 9
- than 2 and $p_1, p_2, ..., p_k$ are distinct random prime numbers, and wherein said ciphertext word C 10
- is a number representative of an encoded form of said message word M that is encoded by 11
- transforming M to said ciphertext word C whereby, 12

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C \equiv M^e \pmod{n},
13
14
               and wherein e is a number relatively prime to (p_1-1), (p_2-1), ..., and (p_k-1);
               said decoding means being operative to perform a decryption process using a decryption
15
       exponent d that is defined by
16
                        d \equiv e^{1/2} \mod ((p_1-1)(p_2-1)...(p_k-1)),
17
               said decryption process including the steps of
18
                        defining a plurality of k sub-tasks in accordance with,
19
                                 \mathbf{M}_{1}' \equiv C_{1}^{d_{1}} \pmod{p_{1}},
20
                                 M_2 \stackrel{}{\searrow} \equiv C_2^{d_2} \pmod{p_2},
21
22
                                 M_k' \equiv C_k^{d_k} \pmod{p_k},
23
                        wherein,
24
25
                                 C_1 \equiv C \pmod{p_1},
                                 C_2 \equiv C \pmod{p_2}
26
27
                                 C_k \equiv C \pmod{p_k}
28
29
30
                                 d_1 \equiv d \pmod{(p_1 - 1)},
                                 d_2 \equiv d \pmod{(p_2 - 1)} and
31
32
                                 d_k \equiv d \pmod{(p_k - 1)}
33
                        solving said sub-tasks to determine results M<sub>1</sub>', M<sub>2</sub>', ... M<sub>k</sub>', and
34
                combining said results of said sub-tasks to produce said receive message word M',
35
36
       whereby M'=M.
                A cyptographic communications system as recited in claim 42 wherein said decoding
       43.
 1
       means is operative to combine said results of said sub-tasks by performing a recursive combining
 2
       process to produce said receive message word M'.
 3
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- 1 44. A cyptographic communications system as recited in claim 41 wherein said decoding
- 2 means is operative to perform said recursive combining process in accordance with
- 3 $Y_i \equiv Y_{i-1} + [(M_i' Y_{i-1}) (w_i^{-1} \mod p_i) \mod p_i] \cdot w_i \mod n$
- 4 wherein $2 \le i \le k$, and
- 5 $M = Y_k \setminus Y_1 = M_1$, and $W_i = \prod_{j \in i} p_j$.
- 1 45. A cyptographic communications system as recited in claim 42 wherein said decoding
- 2 means is operative to combine said results of said sub-tasks by performing a summation process
- 3 to produce said receive message word M'.
- 1 46. A cyptographic communications system as recited in claim 45 wherein said decoding
- 2 means is operative to perform said summation process in accordance with
- $M' \equiv \sum_{i=1}^{k} M'_{i} (w_{i}^{-1} \bmod p_{i}) w_{i} \bmod n,$
- where
- $\mathbf{w}_{i} = \prod_{j \neq i} p_{j}.$
- 1 47. (Three Times Amended) A method for generating a digital signature, comprising the step
- 2 of:
- 3 signing a plaintext message word M to create a signed ciphertext word C, wherein M
- 4 corresponds to a number representative of a message, and
- $0 \le M \le n-1,$
- n being a composite number formed from the product of p₁•p₂•...•p_k, wherein k is an integer
- greater than 2 and $p_1, p_2, ..., p_k$ are distinct random prime numbers, and wherein the signed
- 8 ciphertext word C is a number representative of a signed form of message word M, wherein
- 9 $C \equiv M^d \pmod{n}$, and
- wherein said step of signing includes the steps of
- defining a plurality of k sub-tasks in accordance with
- $C_1 \equiv M_1^{d_1} \pmod{p_1},$

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C_2 \equiv M_2^{d_2} \pmod{p_2},
13
14
                                C_k \equiv M_k^{d_k} \pmod{p_k},
15
                        where
16
                                 M_1 \equiv M \pmod{p_1},
17
                                M_2 \not\equiv M \pmod{p_2},
18
19
                                 M_k \equiv M \pmod{p_k}
20
21
                                 d_1 \equiv d \pmod{(p_1 - 1)},
22
                                 d_2 \equiv d \pmod{(p_2 - 1)}, and
23
25
                                 d_k \equiv d \pmod{(p_k - 1)},
                        wherein d is defined by
26
                                 d = e^{-1} \mod ((p_1 - 1) \cdot (p_2 - 1) \cdot ... \cdot (p_k - 1)), and
27
                                 e is a number relatively prime to (p_1-1), (p_2-1), ..., and (p_k-1),
28
                solving said sub-tasks to determine results C1, C2, ... Ck, and
29
                combining said results of said sub-tasks to produce said ciphertext word C.
30
                A method as recited in claim 47 wherein said step of combining said results of said sub-
 1
       48.
       tasks includes a step of performing a recursive combining process to produce said ciphertext
 2
       word C.
 3
                A method as recited in claim 48 wherein said recursive combining process is performed
 1
       49.
       in accordance with
 2
                        Y_i \equiv Y_{i-1} + [(C_i - Y_{i-1}) (w_i^{-1} \mod p_i) \mod p_i] \bullet v_i \mod n,
 3
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wherein $2 \le i \le k$, and

5
$$C = Y_k, Y_1 = C_1, \text{ and } W_i = \prod_{j < i} p_j.$$

- A method as recited in claim 47 wherein said step of combining said results of said sub-50.
- tasks includes a step of performing a summation process to produce said signed ciphertext word 2
- 3 C.

- A method as recited in claim 50 wherein said summation process is performed in 51. 1
- 2 accordance with

3
$$C \equiv \sum_{i=1}^{k} C_{i} (w_{i}^{-1} \mod p_{i}) w_{i} \mod n,$$
4 where
5
$$w_{i} = \prod_{j \neq i} p_{j}.$$

$$\mathbf{w}_{\mathbf{i}} = \prod_{\mathbf{j} \neq \mathbf{i}} p_{\mathbf{j}}.$$

- (Three Times Amended) A digital signature generation system, comprising: 52.
- a communication medium; 2
- digital signature generating means coupled to said communication medium and operative 3
- to transform a transmit message word M to a signed ciphertext word C, and to transmit said 4
- signed ciphertext word C on said medium, wherein M corresponds to a number representative of 5
- 6 a message, and
- $0 \le M \le n-1$, 7
- n being a composite number formed from the product of $p_1 \cdot p_2 \cdot ... \cdot p_k$ wherein k is an integer 8
- greater than 2 and p1, p2, ..., pk, are distinct random prime numbers, and wherein the signed 9
- ciphertext word C is a number representative of a signed form of said message word M, wherein 10
- $C \equiv M^d \pmod{n}$, 11
- said digital signature generating means being operative to transform said transmit 12
- message word M to said signed ciphertext word C by performing a digital signature generating `13
- process comprising the steps of, 14
- defining a plurality of k sub-tasks in accordance with, 15

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16
17
18
19
                         where,
20
                                 M_1 \equiv M \pmod{p_1},
21.
                                  M_2 \equiv M \pmod{p_2},
22
23
                                  M_k \equiv M \pmod{p_k},
24
25
                                  d_1 \equiv d \pmod{(p - 1)},
26
                                  d_2 \equiv d \pmod{(p_2 \setminus 1)}, and
27
28
                                  d_k \equiv d \pmod{(p_k - 1)}
29
                         wherein d is defined by,
30
                                  d \equiv e^{-1} \mod ((p_1 - 1) \cdot (p_2 - 1) \cdot \dots \cdot (p_k - 1)), and
 31
                                  e is a number relatively prime to (p_1-1), (p_2-1), ..., and (p_k-1),
 32
                 solving said sub-tasks to determine results C1, C2, ... Ck, and
 33
                 combining said results of said sub-tasks to produce said signed ciphertext word C.
 34
```

- 1 53. A digital signature generation system as recited in claim 52 wherein said signature
- 2 generating means is operative to combine said results of said sub-tasks by performing a recursive
- 3 combining process to produce said signed ciphertext word &
- 1 54. A digital signature generation system as recited in claim 53 wherein said digital signature
- 2 generating means is operative to perform said recursive combining process in accordance with

3
$$Y_i \equiv Y_{i-1} + [(C_i - Y_{i-1}) (w_i^{-1} \mod p_i) \mod p_i] \cdot w_i \mod p_i$$

4 wherein
$$2 \le i \not k$$
, and

$$C = Y_k$$
, $Y_i = C_i$, and $W_i = \prod_{j \le i} p_j$.

- A digital signature generation system as recited in claim 52 wherein said signature 55. 1
- generating means is operative to combine said results of said sub-tasks by performing a 2
- summation process to produce said signed message word C. 3
- A digital signature system as recited in claim 55 wherein said signature generating means 56. 1
- is operative to perform said summation process in accordance with 2

$$C \equiv \sum_{i=1}^{k} C_i (w_i^{-1} \mod p_i) w_i \mod n,$$
where

1

$$\mathbf{w_i} = \prod_{j \neq i} p_j.$$

- (Three Times Amended) A digital signature process, comprising the steps of: 57.
- signing a plaintext message word M to create a signed ciphertext word C, wherein M 2
- 3 corresponds to a number representative of a message and wherein

$$4 0 \le M \le n-1$$

- wherein n is a composite number formed by the product of p₁•p₂•...•p_k, k is an integer 5
- greater than 2 and p₁, p₂, ..., p_k are distinct random prime numbers, C is a number 6
- representative of a signed form of message word M, and wherein said encoding step 7
- comprises transforming said message word M to said ciphertext word C whereby, 8

9
$$C \equiv M^d \pmod{n}$$
,

10 wherein d is defined by

$$d \equiv e^{-1} \mod ((p_1 - 1) \cdot (p_2 - 1) \cdot ... \cdot (p_k - 1)), \text{ and }$$

- e is a number relatively prime to (p_1-1) , (p_2-1) , ..., and (p_k-1) ; and 12
- verifying said ciphertext word C to a receive message word M' by performing the steps 13
- 14 of,

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defining a plurality of k sub-tasks in accordance with
15
                                         M_1' \equiv C_1^{e_1} \pmod{p_1},
16
                                          M_2' \equiv C_2^{e_2} \pmod{p_2},
17
18
                                          M_k' \equiv C_k^{e_k} \pmod{p_k},
19
                                 wherein
20
                                          C_1 \not\equiv C \pmod{p_1},
21
                                          C_2 \equiv C \pmod{p_2},
22
23
                                          C_k \equiv C \pmod{p_k},
24
25
                                          e_1 \equiv e \pmod{(p_1 - 1)},
26
                                          e_2 \equiv e \pmod{(p_2 - 1)}, and
27
28
                                           e_k \equiv e \pmod{(p_k - 1)}
29
                         solving said sub-tasks to determine results M<sub>1</sub>', M<sub>2</sub>', ... M<sub>k</sub>', and
30
                combining said results of said sub-tasks to produce said receive message word M',
31
        whereby M'=M.
32
                A digital signature process as recited in claim 57 wherein said step of combining said
        58.
  1
        results of said sub-tasks includes a step of performing\a recursive combining process to produce
  2
        said receive message word M'.
  3
                A digital signature process as recited in claim 58 wherein said recursive combining
        59.
  1
        process is performed in accordance with
  2
                         Y_i \equiv Y_{i-1} + [(M_i' - Y_{i-1}) (w_i^{-1} \mod p_i) \mod p_i] \cdot w_i \pmod n
  3
```

wherein $2 \le i \le k$, and

5
$$M = Y_k$$
, $Y_i = M_i$, and $W_i = \prod_{j < i} p_j$.

- A digital signature process as recited in claim 58 wherein said step of combining said 60. 1
- results of said sub-tasks includes a step of performing a summation process to produce said 2
- receive message word M'. 3
- A digital signature process as recited in claim 60 wherein said summation process is 61. 1
- performed in accordance with 2

3
$$M' \equiv \sum_{i=1}^{k} M'_{i} (w_{i}^{-1} \mod p_{i}) w_{i} \mod n,$$
4 where

$$W_i = \prod_{j \neq i} p_j$$
.

- 62. (Three Times Amended) A digital signature system, comprising:
- a communication medium; 2
- digital signature generating means coupled to said communication medium and adapted 3
- for transforming a message word M to a signed ciphertext word C and for transmitting said 4
- signed ciphertext word C on said medium, wherein M corresponds to a number representative of 5
- a message, and 6

- $0 \le M \le n-1$, wherein n is a composite number of the form 7
- 8 $n = p_1 \cdot p_2 \cdot \ldots \cdot p_k$
- wherein k is an integer greater than 2 and $p_1, p_2, ..., p_k$ are distinct random prime 9
- numbers, and wherein said signed ciphertext word C corresponds to a number representative of a 10
- signed form of said message word M and corresponds to 11
- $C \equiv M^d \pmod{n}$, 12
- wherein d is defined by 13
- $d = e^{-1} \mod ((p_1 1) \cdot (p_2 1) \cdot \dots \cdot (p_k 1)),$ and 14
- e is a number relatively prime to (p_1-1) , (p_2-1) , ..., and (p_k-1) ; and 15

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digital signature verification means communicatively coupled with said communication 16 medium for receiving said signed ciphertext word C via said medium, and being operative to 17 verify said signed ciphertext word C by performing the steps of, 18 defining a plurality of k sub-tasks in accordance with 19 20 21 22 M_k^{λ} $\equiv C_k^{e_k} \pmod{p_k}$ 23 24 wherein 25 $C_1 \equiv C \pmod{p_1}$, $C_2 \equiv C \pmod{p_2}$, 26 27 28 $C_k \equiv C \pmod{p_k}$ 29 $e_1 \equiv e \pmod{(p_1 - 1)},$ 30 $e_2 \equiv e \pmod{(p_2-1)},$ 31 32 $e_k \equiv e \pmod{(p_k \setminus 1)},$ 33 solving said sub-tasks to determine results M₁', M₂', ... M_k', and 34 combining said results of said sub-tasks to produce said receive message word M'

- A digital signature system as recited in claim 62 wherein said decoding means is 1 63.
- operative to combine said results of said sub-tasks by performing a recursive combining process 2
- 3 to produce said receive message word M'.
- A digital signature system as recited in claim 63 wherein said decoding means is 1 64.
- operative to perform said recursive combining process in accordance with 2

wherein M'=M.

35

- $Y_i \equiv Y_{i-1} + (M_i' Y_{i-1}) (w_i^{-1} \mod p_i) \mod p_i] \cdot w_i \mod n,$ 3
- wherein $2 \le i \le k$, and 4
- $M = Y_k$, $Y_1 = M_1$, and $W_i = \prod_{j \in i} p_j$. 5
- A digital signature system as recited in claim 62 wherein said decoding means is 65. 1
- operative combine said results of said sub-tasks by performing a summation process to produce 2
- said receive message word M'. 3
- A digital signature system as recited in claim 65 wherein said decoding means is 1 66.
- operative to perform said summation process accordance with
- $\mathbf{M}' \equiv \sum_{i=1}^{K} \mathbf{M}'_{i} (w_{i}^{-1} \bmod p_{i}) w_{i} \bmod n,$ where
- - A method as recited in claim 17 wherein said step of solving said sub-tasks includes processing each of said sub-tasks by an associated one of a plurality of exponentiator units operating substantially simultaneously.
- A method as recited in claim 17 wherein each of said distinct random prime number has 74. 1
- 2 the same number of bits.
- A cryptographic communications system as recited in claim 22 wherein said step of 75. 1
- solving said sub-tasks includes processing each of said sub-tasks by an associated one of a 2
- plurality of exponentiator units operating substantially simultaneously. 3
- A cryptographic communications system as recited in claim 22 wherein each of said 76.

2 distinct random prime number has the same number of bits.

- 1 77.\ A method as recited in claim 27 wherein said step of solving said sub-tasks includes
- 2 processing each of said sub-tasks by an associated one of a plurality of exponentiator units
- 3 operating substantially simultaneously.
- 1 78. A method as recited in claim 27 wherein each of said distinct random prime number has
- 2 the same number of bits.
- 1 79. A cryptographic communications system as recited in claim 32 wherein said step of
- 2 solving said sub-tasks includes processing each of said sub-tasks by an associated one of a
- 3 plurality of exponentiator units operating substantially simultaneously.
- 1 80. A cryptographic communications system as recited in claim 32 wherein each of said
- 2 distinct random prime number has the same number of bits.
- 1 81. A method as recited in claim 37 wherein said step of solving said sub-tasks includes
- 2 processing each of said sub-tasks by an associated one of a plurality of exponentiator units
- 3 operating substantially simultaneously.
- 1 82. A method as recited in claim 37 wherein each of said distinct random prime number has
- 2 the same number of bits.
- 1 83. A cryptographic communications system as recited in claim 42 wherein said step of
- 2 solving said sub-tasks includes processing each of said sub-tasks by an associated one of a
- 3 plurality of exponentiator units operating substantially simultaneously.
- 1 84. A cryptographic communications system as recited in claim 42 wherein each of said
- 2 distinct random prime number has the same number of bits.

- 1 85. A method as recited in claim 47 wherein said step of solving said sub-tasks includes
- 2 processing each of said sub-tasks by an associated one of a plurality of exponentiator units
- 3 operating substantially simultaneously.
- 1 86. A method as recited in claim 47 wherein each of said distinct random prime number has
- 2 the same number of bits.
- 1 87. A digital signature generation system as recited in claim 52 wherein said step of solving
- 2 said sub-tasks includes processing each of said sub-tasks by an associated one of a plurality of
- 3 exponentiator units operating substantially simultaneously.
- 1 88. A digital signature generation system as recited in claim 52 wherein each of said distinct
- 2 random prime number has the same number of bits.
- 1 89. A digital signature process as recited in claim 57 wherein said step of solving said sub-
- 2 tasks includes processing each of said sub-tasks by an associated one of a plurality of
- 3 exponentiator units operating substantially simultaneously.
- 1 90. A digital signature process as recited in claim 57 wherein each of said distinct random
- 2 prime number has the same number of bits.
- 1 91. A digital signature system as recited in claim 62 wherein said step of solving said sub-
- 2 tasks includes processing each of said sub tasks by an associated one of a plurality of
- 3 exponentiator units operating substantially simultaneously.
- 1 92. A digital signature system as recited in claim 62 wherein each of said distinct random
- 2 prime number has the same number of bits.